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# QCD Radiative Corrections to the Leptonic Decay Rate of the $B_c$ Meson

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## Abstract

The QCD radiative corrections to the leptonic decay rate of the  $B_c$  meson are calculated using the formalism of nonrelativistic QCD (NRQCD) to separate short-distance and long-distance effects. The  $B_c$  decay constant is factored into a sum of NRQCD matrix elements each multiplied by a short-distance coefficient. The short-distance coefficient for the leading matrix element is calculated to order  $\alpha_s$  by matching a perturbative calculation in full QCD with the corresponding perturbative calculation in NRQCD. This short-distance correction decreases the leptonic decay rate by approximately 15%.

The study of heavy quarkonium systems has played an important role in the development of quantum chromodynamics (QCD). Some of the earliest applications of perturbative QCD were calculations of the decay rates of charmonium [1]. These calculations were based on the assumption that, in the nonrelativistic limit, the decay rate factors into a short-distance perturbative part associated with the annihilation of the heavy quark and antiquark and a long-distance part associated with the quarkonium wavefunction. This simple factorization assumption fails for P-wave states [2], which satisfy a more general factorization formula containing two nonperturbative factors [3]. Calculations of the annihilation decay rates of heavy quarkonium have recently been placed on a solid theoretical foundation by Bodwin, Braaten, and Lepage [4]. Their approach is based on nonrelativistic QCD (NRQCD), an effective field theory that is equivalent to QCD to any given order in the relative velocity  $v$  of the heavy quark and antiquark [5]. Using NRQCD to separate the short-distance and long-distance effects, Bodwin, Braaten, and Lepage derived a general factorization formula for the inclusive annihilation decay rates of heavy quarkonium. The short-distance factors in the factorization formula can be calculated using perturbative QCD, and the long-distance factors are defined rigorously in terms of matrix elements of NRQCD that can be evaluated using lattice calculations. The general factorization formula applies equally well to S-waves, P-waves, and higher orbital-angular-momentum states, and it can be used to systematically incorporate relativistic corrections to the decay rates.

Since the top quark decays too quickly to produce narrow resonances, the only heavy quark-antiquark bound states that remain to be discovered are the  $\bar{b}c$  mesons and their antiparticles. The possibility that  $\bar{b}c$  mesons may be discovered at existing accelerators has stimulated much recent work on the properties of these mesons [6] and on their production cross sections at high energy colliders [7]. Once produced, a  $\bar{b}c$  meson will cascade down through lower energy  $\bar{b}c$  states via hadronic or electromagnetic transitions to the pseudoscalar ground state  $B_c$  which decays weakly. The discovery of the  $B_c$  meson will require a detailed understanding of its decay modes. In this paper, we compute the short-distance QCD radiative correction to the leptonic decay rate of the  $B_c$ . We use the formalism of NRQCD to factor the amplitude for the decay into short-distance coefficients multiplied by NRQCD matrix elements. The short-distance coefficient for the leading matrix element is calculated

to next-to-leading order in  $\alpha_s$  by matching a perturbative calculation in full QCD with the corresponding perturbative calculation in NRQCD.

The leptonic decay of the  $B_c$  proceeds through a virtual  $W^+$  as in Figure 1. The  $W^+$  couples to the  $B_c$  through the axial-vector part of the charged weak current. All QCD effects, both perturbative and nonperturbative, enter into the decay rate through the decay constant  $f_{B_c}$ , defined by the matrix element

$$\langle 0 | \bar{b} \gamma^\mu \gamma_5 c | B_c(P) \rangle = i f_{B_c} P^\mu, \quad (1)$$

where  $|B_c(P)\rangle$  is the state consisting of a  $B_c$  with four-momentum  $P$ . It has the standard covariant normalization  $\langle B_c(P') | B_c(P) \rangle = (2\pi)^3 2P^0 \delta^3(P' - P)$  and its phase has been chosen so that  $f_{B_c}$  is real and positive. In terms of the decay constant  $f_{B_c}$ , the leptonic decay rate is

$$\Gamma(B_c \rightarrow \ell^+ \nu_\ell) = \frac{1}{8\pi} |V_{bc}|^2 G_F^2 M_{B_c} f_{B_c}^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{M_{B_c}^2}\right)^2, \quad (2)$$

where  $V_{bc}$  is the appropriate Kobayashi-Maskawa matrix element,  $G_F$  is the Fermi constant,  $M_{B_c}$  is the mass of the  $B_c$  meson, and  $m_\ell$  is the mass of the charged lepton.

The formula (1) provides a nonperturbative definition of the decay constant  $f_{B_c}$ , so that it can be calculated using lattice QCD simulations. One of the difficulties with such a calculation is that it requires a lattice with large volume and fine lattice spacing, since the strong interactions must be accurately simulated over many distance scales. The long-distance scales range from  $1/\Lambda_{QCD}$ , the scale of nonperturbative effects associated with gluons and light quarks, to the scale  $1/(m_c v)$  of the meson structure, where  $v$  is the typical relative velocity of the charm quark. The short-distance scales include the Compton wavelengths  $1/m_c$  and  $1/m_b$  of the heavy quark and antiquark. A more effective strategy for calculating  $f_{B_c}$  is to separate short-distance effects from long-distance effects, to calculate the short distance effects analytically using perturbation theory in  $\alpha_s$ , and to use lattice simulations only for calculating the long-distance effects. Having already taken into account the short-distance effects, one can use a much coarser lattice which provides enormous savings in computer resources.

An elegant way to separate short-distance and long-distance effects is to use NRQCD,

an effective field theory in which heavy quarks are described by a nonrelativistic Schroedinger field theory of 2-component Pauli spinors. The lagrangian is

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{light}} + \psi_c^\dagger \left( iD_0 + \mathbf{D}^2/(2m_c) \right) \psi_c + \chi_b^\dagger \left( iD_0 - \mathbf{D}^2/(2m_b) \right) \chi_b + \dots, \quad (3)$$

where  $\mathcal{L}_{\text{light}}$  is the usual relativistic lagrangian for gluons and light quarks. The 2-component field  $\psi_c$  annihilates charm quarks, while  $\chi_b$  creates bottom antiquarks. The typical velocity  $v$  of the charm quark in the meson provides a small parameter that can be used as a non-perturbative expansion parameter. Lattice simulations using the terms given explicitly in (3) can be used to calculate matrix elements for the  $B_c$  with errors of order  $v^2$ . To obtain higher accuracy, additional terms, represented by  $\dots$  in (3), must be included. There are 8 terms that must be added to decrease the errors to order  $v^4$ .

To express the decay constant  $f_{B_c}$  in terms of NRQCD matrix elements, we must express the axial-vector current  $\bar{b}\gamma^\mu\gamma_5 c$  in terms of NRQCD fields. Only the  $\mu = 0$  component contributes to the matrix element (1) in the rest frame of the  $B_c$ . This component of the current has an operator expansion in terms of NRQCD fields:

$$\bar{b}\gamma^0\gamma_5 c = C_0(m_b, m_c) \chi_b^\dagger \psi_c + C_2(m_b, m_c) (\mathbf{D}\chi_b)^\dagger \cdot \mathbf{D}\psi_c + \dots, \quad (4)$$

where  $C_0$  and  $C_2$  are short-distance coefficients that depend on the quark masses  $m_b$  and  $m_c$ . By dimensional analysis, the coefficient  $C_2$  is proportional to  $1/m_Q^2$ . The contribution to the matrix element  $\langle 0 | \bar{b}\gamma^0\gamma_5 c | B_c \rangle$  from the operator  $(\mathbf{D}\chi_b)^\dagger \cdot \mathbf{D}\psi_c$  is suppressed by  $v^2$  relative to the operator  $\chi_b^\dagger \psi_c$ , where  $v$  is the typical velocity of the charm quark in the  $B_c$ . The  $\dots$  in (4) represent other operators whose contributions are suppressed by higher powers of  $v^2$ .

The short-distance coefficient  $C_0$  and  $C_2$  can be determined by matching perturbative calculations of the matrix elements in full QCD and NRQCD. A convenient choice for matching is the matrix element between the vacuum and the state  $|c\bar{b}\rangle$  consisting of a  $c$  and a  $\bar{b}$  on their perturbative mass shells with nonrelativistic four-momenta  $p$  and  $p'$  in the center of momentum frame:  $\mathbf{p} + \mathbf{p}' = 0$ . The matching condition is

$$\langle 0 | \bar{b}\gamma^0\gamma_5 c | c\bar{b} \rangle \Big|_{\text{P-QCD}} = C_0 \langle 0 | \chi_b^\dagger \psi_c | c\bar{b} \rangle \Big|_{\text{P-NRQCD}} + C_2 \langle 0 | (\mathbf{D}\chi_b)^\dagger \cdot \mathbf{D}\psi_c | c\bar{b} \rangle \Big|_{\text{P-NRQCD}} + \dots, \quad (5)$$

where P-QCD and P-NRQCD represent perturbative QCD and perturbative NRQCD, respectively. At leading order in  $\alpha_s$ , the matrix element on the left side of (5) is  $\bar{v}_b(-\mathbf{p})\gamma^0\gamma_5 u_c(\mathbf{p})$ . The Dirac spinors are

$$u_c(\mathbf{p}) = \sqrt{\frac{E_c + m_c}{2E_c}} \begin{pmatrix} \xi \\ \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{E_c + m_c} \xi \end{pmatrix}, \quad (6)$$

$$v(-\mathbf{p}) = \sqrt{\frac{E_b + m_b}{2E_b}} \begin{pmatrix} \frac{(-\mathbf{p})\cdot\boldsymbol{\sigma}}{E_b + m_b} \eta \\ \eta \end{pmatrix}, \quad (7)$$

where  $\xi$  and  $\eta$  are 2-component spinors and  $E_Q = m_Q^2 + \mathbf{p}^2$ . Making a nonrelativistic expansion of the spinors to second order in  $\mathbf{p}/m_Q$ , we find

$$\bar{v}_b(-\mathbf{p})\gamma^0\gamma_5 u_c(\mathbf{p}) \approx \eta_b^\dagger \xi_c \left( 1 - \frac{1}{8} \left( \frac{m_b + m_c}{m_b m_c} \right)^2 \mathbf{p}^2 + \dots \right). \quad (8)$$

At leading order in  $\alpha_s$ , the matrix elements on the right side of (5) are  $\eta_b^\dagger \xi_c$  and  $\mathbf{p}^2 \eta_b^\dagger \xi_c$ . The short distance coefficients are therefore  $C_0 = 1$  and

$$C_2 = -\frac{1}{8 m_{\text{red}}^2}, \quad (9)$$

where  $m_{\text{red}} = m_b m_c / (m_b + m_c)$  is the reduced mass.

To determine the short distance coefficients to order  $\alpha_s$ , we must calculate the matrix elements on both sides of (5) to order  $\alpha_s$ . We will calculate the order- $\alpha_s$  correction only for the coefficient  $C_0$ , since the contribution proportional to  $C_2$  is suppressed by  $v^2$ . The coefficient  $C_0$  can be isolated by taking the limit  $\mathbf{p} \rightarrow 0$ , in which case the matrix element of  $(\mathbf{D}\chi_b)^\dagger \cdot \mathbf{D}\psi_c$  vanishes. We first calculate the matrix element on the left side of (5) to order  $\alpha_s$ . The relevant diagrams are shown in Fig. 2. The tree diagram in Fig. 2a gives a product of Dirac spinors  $\bar{v}_b \gamma_0 \gamma_5 u_c$ . We calculate the loop diagrams in Fig. 2b-2d in Feynman gauge, using dimensional regularization to regularize both infrared and ultraviolet divergences. Momentum integrals are analytically continued to  $D = 4 - 2\epsilon$  spacetime dimensions, requiring the introduction of a regularization scale  $\mu$ . The effects of the quark self-energy diagrams in Fig. 2b and 2c is to multiply the matrix element by the renormalization constants  $\sqrt{Z_b}$  and  $\sqrt{Z_c}$ , where

$$\sqrt{Z_Q} = 1 + \frac{2\alpha_s}{3\pi} \left( -\frac{1}{4} \frac{1}{\epsilon_{UV}} - \frac{1}{2} \frac{1}{\epsilon_{IR}} - \frac{3}{4} (\log 4\pi - \gamma) + \frac{3}{2} \log \frac{m_Q}{\mu} - 1 \right). \quad (10)$$

The subscript *UV* or *IR* on the  $\epsilon$ 's indicates whether the divergence is of ultraviolet or infrared origin. Combining the propagators using the Feynman parameter trick and using the fact that the external quarks  $\bar{b}$  and  $c$  are on-shell, the vertex correction from Fig. 2d can be reduced to the tree-level diagram in Fig. 2a multiplied by the factor

$$\begin{aligned} \Lambda = & \frac{64\pi i\alpha_s}{3} \int_0^1 dx \int_0^{1-x} dy \mu^{2\epsilon} \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2 - (xp' - yp)^2 + i\epsilon]^3} \\ & \left[ 2(1-x-y)p \cdot p' + xm_c^2 + ym_b^2 + (x+y)m_b m_c \right. \\ & \left. + 2(1-\epsilon)(xm_c - ym_b)^2 - (1-\epsilon)(xp' - yp)^2 - \frac{(1-\epsilon)^2}{(2-\epsilon)} k^2 \right]. \quad (11) \end{aligned}$$

After integrating over  $k$ , it is convenient to change variables from the Feynman parameters to  $s = x + y$  and  $t = x/s$ . The  $s$ -integrals are trivial, but the  $t$ -integrals must be evaluated with care using the  $i\epsilon$  prescription in the denominator. Setting  $p \cdot p' \approx m_b m_c (1 + v^2/2)$ , where  $v$  is the relative velocity of the  $\bar{b}$  and  $c$ , and taking the limit  $v \rightarrow 0$ , the integral reduces to

$$\begin{aligned} \Lambda = & \frac{2\alpha_s}{3\pi} \left[ \frac{1}{2} \frac{1}{\epsilon_{UV}} + \frac{1}{\epsilon_{IR}} + \frac{3}{2} (\log 4\pi - \gamma) + \frac{\pi^2}{v} - 3 \frac{m_b}{m_b + m_c} \log \frac{m_c}{\mu} - 3 \frac{m_c}{m_b + m_c} \log \frac{m_b}{\mu} - 1 \right. \\ & \left. - \frac{i\pi}{v} \left( \frac{1}{\epsilon_{IR}} - 2 \log \frac{2m_{\text{red}}v}{\mu} + \log 4\pi - \gamma \right) \right], \quad (12) \end{aligned}$$

where  $m_{\text{red}} = m_b m_c / (m_b + m_c)$ . Multiplying the tree-level matrix element by the vertex factor  $1 + \Lambda$  and by the renormalization constants  $\sqrt{Z_b}$  and  $\sqrt{Z_c}$ , we obtain the final answer for the matrix element to order  $\alpha_s$ :

$$\begin{aligned} \langle 0 | \bar{b} \gamma^0 \gamma_5 c | \bar{c} b \rangle \Big|_{\text{P-QCD}} = & \bar{v}_b \gamma^0 \gamma^5 u_c \left\{ 1 + \frac{2\alpha_s}{3\pi} \left[ \frac{\pi^2}{v} + \frac{3}{2} \frac{m_b - m_c}{m_b + m_c} \log \frac{m_b}{m_c} - 3 \right. \right. \\ & \left. \left. - \frac{i\pi}{v} \left( \frac{1}{\epsilon_{IR}} - 2 \log \frac{2m_{\text{red}}v}{\mu} + \log 4\pi - \gamma \right) \right] \right\}. \quad (13) \end{aligned}$$

Note that the ultraviolet divergences have cancelled. The imaginary part of (13) arises because it is possible for the quark and antiquark created by the current to scatter on-shell.

We next compute the NRQCD matrix element on the right side of (5) to order  $\alpha_s$ . The relevant Feynman diagrams are again shown in Fig. 2. The tree diagram in Fig. 2a gives

a product of Pauli spinors  $\eta_b^\dagger \xi_c$ . It is convenient to calculate the loop diagrams in Coulomb gauge. In this gauge, the coupling of transverse gluons to heavy quarks is proportional to the heavy quark velocity, and therefore does not contribute in the limit  $v \rightarrow 0$ . We therefore need only calculate the contribution from Coulomb exchange. The wavefunction renormalization factors associated with the diagrams in Fig. 2b and Fig. 2c are trivial:  $\sqrt{Z_b} = \sqrt{Z_c} = 1$ . The diagram in Fig. 2d reduces to a multiplicative correction to the tree-level matrix element from Fig. 2a:

$$\Lambda = -\frac{16\pi i \alpha_s}{3} \mu^{2\epsilon} \int \frac{d^D q}{(2\pi)^D} \frac{1}{\mathbf{q}^2} \frac{1}{E_b + q_0 - (\mathbf{p} + \mathbf{q})^2/2m_b + i\epsilon} \frac{1}{E_c - q_0 - (\mathbf{p} + \mathbf{q})^2/2m_c + i\epsilon}, \quad (14)$$

where  $E_Q = p^2/2m_Q$ . The infrared-divergent integral has been dimensionally regularized. After using contour integration to integrate over the energy  $q_0$  of the exchanged gluon, we find that (14) reduces to an integral over the gluon's three-momentum:

$$\Lambda = \frac{32\pi\alpha_s m_{\text{red}}}{3} \mu^{2\epsilon} \int \frac{d^{3-2\epsilon} q}{(2\pi)^{3-2\epsilon}} \frac{1}{\mathbf{q}^2} \frac{1}{\mathbf{q}^2 + 2\mathbf{p} \cdot \mathbf{q} - i\epsilon}. \quad (15)$$

Evaluating the regularized integral in (15), we obtain

$$\langle 0 | \chi_b^\dagger \psi_c | c \bar{b} \rangle \Big|_{\text{P-NRQCD}} = \eta_b^\dagger \xi_c \left\{ 1 + \frac{2\alpha_s}{3\pi} \left[ \frac{\pi^2}{v} - \frac{i\pi}{v} \left( \frac{1}{\epsilon_{IR}} - 2 \log \frac{2m_{\text{red}} v}{\mu} + \log 4\pi - \gamma \right) \right] \right\}, \quad (16)$$

where  $v$  is the relative velocity of the  $\bar{b}$  and  $c$ :  $|\mathbf{p}| = m_{\text{red}} v$ . Note that the infrared divergences and the factors of  $1/v$  are identical to those in the matrix element (13) for full QCD.

Comparing the matrix elements (13) and (16), we can read off the short-distance coefficient  $C_0$  from the matching condition (5):

$$C_0 = 1 + \frac{\alpha_s(m_{\text{red}})}{\pi} \left[ \frac{m_b - m_c}{m_b + m_c} \log \frac{m_b}{m_c} - 2 \right]. \quad (17)$$

We have chosen  $m_{\text{red}}$  as the scale of the running coupling constant. To the accuracy of this calculation, any scale of order  $m_b$  or  $m_c$  would be equally correct. Setting  $m_b = 4.5$  GeV and  $m_c = 1.5$  GeV, we find that  $\alpha_s(m_{\text{red}}) \approx 0.34$ , and  $C_0 \approx 0.85$ .

Our final result for the decay constant of the  $B_c$  is

$$if_{B_c} M_{B_c} = C_0 \langle 0 | \chi_b^\dagger \psi_c | B_c \rangle + C_2 \langle 0 | (\mathbf{D}\chi_b)^\dagger \cdot \mathbf{D}\psi_c | B_c \rangle. \quad (18)$$

The short-distance coefficient  $C_0$  is given to next-to-leading order in  $\alpha_s$  in (17), while  $C_2$  is given to leading order in (9). The uncertainties consist of the perturbative errors in the short distance coefficients and an error of relative order  $v^4$  from the neglect of matrix elements that are higher order in  $v^2$ . To achieve an error of order  $v^4$ , the matrix element  $\langle 0 | (\mathbf{D}\chi_b)^\dagger \cdot \mathbf{D}\psi_c | B_c \rangle$  can be calculated by lattice simulations using those terms in the NRQCD lagrangian that are given explicitly in (3), but  $\langle 0 | \chi_b^\dagger \psi_c | B_c \rangle$  must be calculated using an improved action. If an error of order  $v^2$  is sufficient accuracy, than the matrix element  $\langle 0 | (\mathbf{D}\chi_b)^\dagger \cdot \mathbf{D}\psi_c | B_c \rangle$  can be dropped, and  $\langle 0 | \chi_b^\dagger \psi_c | B_c \rangle$  can be calculated using only those terms in the NRQCD lagrangian that are given explicitly in (3). The parameters in this lagrangian are  $\alpha_s$  and the quark masses  $m_b$  and  $m_c$ , all of which can be tuned so as to reproduce the spectroscopy of charmonium and bottomonium. These simulations should be very accurate for the intermediate case of the  $\bar{b}c$  system. In the absence of any lattice calculations of the  $\bar{b}c$  system, the matrix element  $\langle 0 | \chi_b^\dagger \psi_c | B_c \rangle$  can be estimated using wavefunctions at the origin from nonrelativistic potential models:

$$|\langle 0 | \chi_b^\dagger \psi_c | B_c \rangle|^2 \approx 2M_{B_c} \frac{3}{2\pi} |R(0)|^2. \quad (19)$$

The factor of  $2M_{B_c}$  takes into account the relativistic normalization of the state  $|B_c\rangle$ .

Although it may not be relevant to the  $B_c$  meson, it is interesting to consider the limit  $m_b \gg m_c$ . In this limit, the perturbative correction in (17) contains a large logarithm of  $m_b/m_c$ . Heavy Quark Effective Theory can be used to sum up the leading logarithms of  $m_b$  [8]:

$$C(\alpha_s) \longrightarrow \left( \frac{\alpha_s(\Lambda)}{\alpha_s(m_b)} \right)^{6\pi/(33-2n_f)} \left\{ 1 + \frac{\alpha_s(m_c)}{\pi} \left[ \log \frac{\Lambda}{m_c} - 2 \right] \right\}, \quad (20)$$

where  $n_f$  is the number of light flavors, including the charm quark, and where  $\Lambda$  is an arbitrary factorization scale.

We have calculated the short-distance QCD radiative correction to the leptonic decay rate of the  $B_c$  meson. This calculation provides an illustration of the factorization methods

based on NRQCD that were developed in Ref. [4]. The decay constant  $f_{B_c}$  of the  $B_c$  was factored into short-distance coefficients multiplied by NRQCD matrix elements. The radiative correction to the coefficient of the leading matrix element decreases the leptonic decay rate of the  $B_c$  meson by about 15%.

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## Figure Captions

1. Diagram for the annihilation of  $B_c$  into lepton pairs via a virtual  $W^+$ . The shaded oval represents the wavefunction of the  $B_c$ .
2. Diagrams for the matrix elements  $\langle 0 | \bar{b} \gamma^0 \gamma_5 c | c \bar{b} \rangle$  in perturbative QCD and  $\langle 0 | \chi_b^\dagger \psi_c | c \bar{b} \rangle$  in perturbative NRQCD.

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